Introduction to area and volume for visually impaired learners using a tactile three dimensional grid

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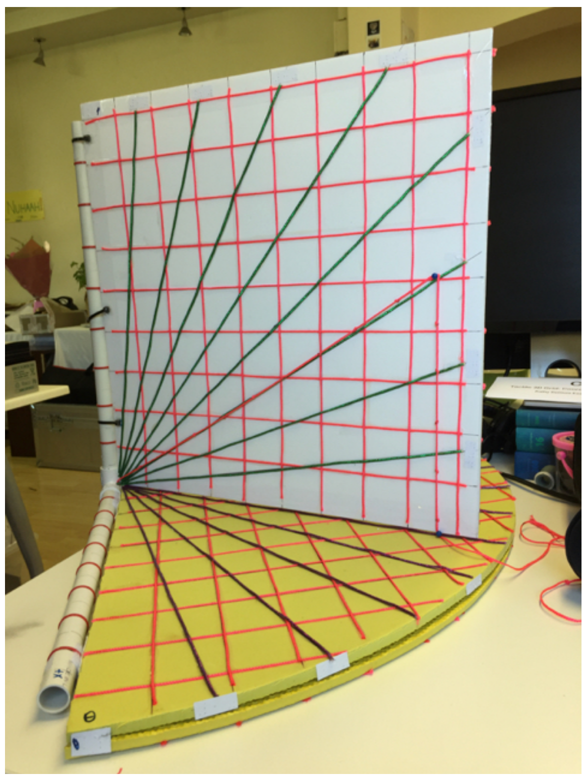
Materials for every two students: a tactile three dimensional grid (CARDIS). Instructions for constructing the CARDIS can be found at astrosense.astro4dev.org. For this exercise, only the simplest version of the CARDIS is needed; the angle markings are not necessary, and neither is the vertical plane. All that is needed are the axes and the xy plane with the grid markings.



Materials per student: A piece of string cut such that the length is an exact number of tickmarks on the grid. A piece of paper cut such that both the length and width are an exact number of tickmarks. A second piece of paper that has the same AREA but different dimensions. [An example of this might be one piece of paper that is 2 units wide by 6 units long, and another piece of paper that is 3 units wide by 4 units long.] A box, preferably with length, width and height all equal to an exact number of tickmarks.

1. Introduction to the grid

Take a little while to explore the grid. You will notice that there is a corner, like the corner of a room, and three “arms”. The shape of the corner is important; think of the corner of a room. There is usually a floor and two walls.

In math, we call the arms “axes”: one axis, two axes, three axes. We usually give each axis a name so that we know which one we are talking about, just like you have your own name! We call them x, y, and z. (You could call them Tumi, Lerato, or John, but then nobody else would know what you were talking about! If we all call them by the same name, we can better communicate about math.)

Find the x-axis, the y-axis, and the z-axis. They are labelled in Braille.

Now explore the flat part. The flat part has the x-axis running down one side, and the y-axis running down the other side, so we call this the xy plane. A “plane” is something perfectly flat, like a piece of paper on a table. This xy plane is like the floor in our corner. If we wanted, we could also make planes that would be like the walls, but right now we are going to work only with the xy plane, the floor.

In math we always start counting from zero. These three axes all have the same “zeropoint”; this is the point from which we start counting. In math, we call it the origin. Find it on your grid. It is the point right inside the corner, where the corner of the floor meets the corners of the two walls.

Find the zeropoint again, and feel along the x-axis. There are markers along the axis that you can feel. These are called “tickmarks”, and they are used to measure a distance along an axis. The special thing about tickmarks is that they are always the same distance apart. We will talk about that more later.

Try counting along the x-axis. Zero is the zeropoint (duh), and One will be the first mark you come to. Two will be the second one, etc. How many tickmarks are there on the x-axis?

Next do the same for the y-axis and the z-axis. Count the tickmarks.

Notice that there are lines running across the floor. Each x “tickmark” has a line starting next to it and stretching across the floor. Each y “tickmark” also has a line starting next to it and stretching across the floor. There are a lot of places where the lines cross; find some of those places.

Let’s finding a specific crossing place. Count four tickmarks along the x-axis, and have one person keep hold of the line starting at the fourth tickmark. Now count three tickmarks along the y-axis, and have the other person keep hold of the line starting at the third tickmark. Now run your hands over those two lines, starting at the tickmarks on the axes and moving out to the place where those lines cross. This crossing-point has a name, too!

The name of this crossing-point is x = 4, y = 3. It is the ONLY point on your floor-grid with that name! Having a name for each crossing-point makes it easy to describe that spot to somebody else. It's like telling someone how to get to your house; you might tell them what street to walk down, and how far to walk before they turn. This is a way to describe to someone else how to find that point on the floor-grid! Walk down the +x-axis for 4 tickmarks, and then turn in the +y direction and walk 3 more tickmarks. We can describe any crossing-place on the floor-grid this way.

Assignment: Each person should find another crossing-place on the floor-grid, and figure out what its name is.

2. Units

What is a unit? A unit is a name for an official "amount" of something. For example, a "meter" is a unit of length. The countries of the world have gotten together and decided exactly how long a meter is. As far back as the year 1799 there was a platinum bar in Paris that represented exactly how long a meter is. Today scientists define a meter a little differently, but they all agree on the definition. Another kind of unit is a "second"; this is a unit of time. It's a little trickier to agree on how long a second is, but scientists have managed to do it.

Why do we need units? Let's say that you want to describe an object to someone who is far away. You cannot give them the object to hold. But you want them to know how long it is, or how heavy it is. You can't use beans to measure out how long it is, because everybody's beans are a different size. You can't use your fingers to measure it out, because everybody's fingers are a different size. Units allow you to describe things using words that everyone shares and agrees on: everyone knows how long a meter is. Everyone knows how heavy a kilogram is. So if you describe the length of your object in units of meters, EVERYONE will know how long it is. If you describe the weight of your object in kilograms, EVERYONE will know how heavy it is.

Most rulers are marked with one kind of units. The most common unit for rulers is centimeters; this usually means that the distance between two tickmarks is exactly one centimeter.

The distance between the tickmarks on the CARDIS are not exactly equal to any official kind of unit. However, all of you should have a grid with the same tickmarks. Each group can name the unit anything you like; do that now! On these instructions we will call them "Charlies", but you don't have to use that name.

3. What are Dimensions?

Take your piece of string. You can always make shapes with your string, but we are going to stretch it out and make it as long as we can, straight like a cane or stick. Stretch it out! Your string really only stretches in one direction. If you were an ant crawling along the string, you could only go forwards and backwards; you couldn't turn right or left! We call this "one dimensional." A dimension represents a direction you are able to move, like walking along a straight street with no other streets that intersect it. On a street you can walk forwards or backwards, but you can't turn right or left.

Measuring in One Dimension
A string only spreads out in one direction, so it is approximately one dimensional. What can we measure about it? The length!

Let's use our grid to measure the length of the string in units of Charlies. To measure, put one end of the string at the origin, and then stretch the string out along one axis. Make sure to hold down the end at the origin, don't let it move. One person can hold the end of the string down, and the other person can count tickmarks along the axis. Count the number of tickmarks along the axis. Then switch, and let the other person count. Make sure you agree on the number! This number is the length of your string in your units. If you counted ten tickmarks, and your units are Charlies, then the length of your string is ten Charlies.

Assignment: Does it matter which axis you use? Do they all work the same way? Try it!

Measuring in Two dimensions
Now take your first piece of paper. What can you measure about this piece of paper?

We need TWO axes at the same time to measure the piece of paper! It doesn't just have length; it also has width. To describe how big the paper is, we have to use two directions. This means that the paper is two dimensional. This is like being able to walk along a street, forwards and backwards, but ALSO being able to turn left or right.

When people talk about length and width, sometimes it is hard to tell which direction is which. For example, some people might use the x-axis for length, and the y-axis for width. Some people might decide that whichever side is longer is the length, and whichever side is shorter is the width. So it can be confusing. But it doesn't really matter, so don't worry about it.

4. Counting Squares
Take your first piece of paper, and lay it down on the flat grid. Make sure that one corner is at the origin; hold it down tight. One person should hold the paper down, and the other person should explore the paper. Feel underneath the paper; can you count the squares that are underneath? Then switch and let the other person hold it down. Take turns counting the squares that are underneath the paper, and then compare your results. If you don't agree, try it again!

Notice that you can also count tickmarks along the edge of the paper. Go ahead and count the number of tickmarks along the x axis, and then the number along the y axis. Did you notice that the tickmarks line up with the edges of the squares underneath the paper?

Let's call the x-axis the length, and the y-axis the width. After measuring the length and width, multiply the two numbers together. For example, if you measured 4 tickmarks in one direction and 5 tickmarks in the other direction, multiply those two numbers together to get 20.

How does the number you get compare to the number of squares you counted under the paper? The number should be the same! You have now measured the SURFACE AREA of the paper. Surface just means the outside of something. The surface of your body is made out of skin. The surface of an orange is made out of orange peel.

The idea of area is a little trickier. Area expresses the size of something that is two-dimensional. In our grid, the paper spreads out in both the x direction and the y direction, so it really only has two dimensions. You can use area to describe the size of a towel or rug, but it doesn't work for a piece of string! String really has only one dimension, so you have to describe it with length, not area.

What are the UNITS of surface area? Well, if the length of your unit is Charlies, it can't be just plain old Charlies. A Charlie is one-dimensional, and area is two-dimensional. If a square is one Charlie long and one Charlie wide, we say that it has an area of one Charlie times one Charlie, or "square Charlies". Square Charlies are two-dimensional. You can only use plain Charlies for length, and you can only use square Charlies for area! If you buy new carpet for your house, you will order it in "square meters". If you a special kind of paper, you might order it in "square centimeters".

However, one square meter of carpet doesn't have to be exactly one meter long by one meter wide, though. Just as with your paper, it might be two meters long and one-half meter wide – but the surface area is still one square meter.

Now take your second piece of paper. Once again take turns holding down the corner of the paper at the origin, and counting the squares underneath the paper. Also measure how many tickmarks the paper covers in the x direction, and then the number in the y direction. Once again, multiply the two numbers together, and the length and the width, and then calculate the area. Once again, the two numbers multiplied together should be equal to the number of squares under the paper.

Hmmm…Are the length and width of this paper the same as the first piece of paper? [Answer should be no.] Is the surface area the same? [Answer should be yes.] It is possible for two surface areas to have different dimensions (lengths) but still have the same surface area!

Assignment: With your partner, think up a THIRD way to cut a piece of paper so that it has the same area as the first two, but not the same length and width.

5. Measuring in Three dimensions

Take your box and put it on the grid so that one corner is at the origin. Let one person hold it down, and the other person measure it by counting along the tickmarks. Notice that in order to describe the box, you have to make measurements in THREE dimensions! The box has length, width, AND height. It spreads out in all three directions, and that's why we call this a three-dimensional object.

Measure the length (x direction), width (y direction), and height of the box (z direction). Take turns, so that everybody gets to measure. Next, count the squares in the xy plane underneath the box.

Let's think about these numbers. You have already figured out that if you multiply the x measurement and the y measurement together that you will get the same number as the number of squares underneath the box. Check your numbers and make sure this is true; if it isn't true, then you have made a mistake. Figure out where you made your mistake!a

How do we describe how big a box is? One good way is to calculate its volume; this is a measurement of how much the box will hold! A big box has a bigger volume than a tiny box, and will hold more stuff. Imagine how many oranges you can put into a box; you can put more oranges inside a big box than inside a tiny box!

To calculate the volume, take the length and width and height, and multiply all three numbers together. Try this with the measurements of your box.

Another way to calculate the volume of a box is to multiply the surface area of the bottom by the height of the box. Try this; the number should come out to be the SAME as it did by multiplying the length and width and height together. If the number didn't come out the same, you made a mistake. Figure out where you made your mistake!

What are the units for volume? It can't be Charlies, or even square Charlies! What would you call the volume of a little box that was one Charlie long, one Charlie wide, and one Charlie tall? This little box would be a "cube", and so we would call it one "cubic Charlie". Isn't that funny? You can describe ONLY volume with cubic Charlies; you can't describe surface area, or length. Only volume.

If you order gravel to build a road, you might order it in cubic meters. When you buy juice at the store, you might buy it in liters, or in milliliters. These are units of volume; a milliliter is just one cubic centimeter of volume. A cubic meter of gravel – no matter whether it is poured out on the ground, or in a wheelbarrow, or in a bin – would fill up a box that is one meter long by one meter wide by one meter tall.

Just as with the two pieces of paper, it is possible to have two boxes which have different dimensions – different length, width, height – but have the same volume.

Assignment: With your partner, think up a second way to build a box so that it has the same volume as your box, but not the same length and width and height.

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