Tactile 3D Grid: Coordinates and Relative Dimension in Space (CARDIS)

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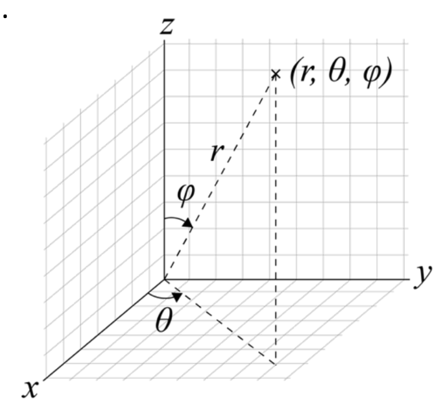
Visualization of complex ideas is particularly difficult for blind and visually impaired learners. We have developed a tactile 3D Cartesian grid that can be made from inexpensive and easily available materials. The CARDIS will also be useful for sighted learners who have trouble visualizing in three dimensions.

There are many possible applications, but here we briefly demonstrate one of the most difficult: the conversions between the Cartesian and Spherical coordinate systems. This is typically encountered in third semester calculus, and introduced as 3D drawings. Both systems describe the coordinates of a point in space represented by the vector **r**.

The introductory 3D drawing usually consists of one octant of the Cartesian system, specifically the octant which represents the section where x, y, and z are all positive. A point is specified within that octant, and the vector is drawn from the origin to the specified point.

The orientation of the axes might be confusing for some students. In two dimensions, students are familiar with x being the abscissa and horizontal, and y being the ordinate and vertical. However, when adding the third dimension, it is necessary to keep the system as right-handed. To understand what this means, stretch out the thumb and forefinger of your **right** hand such that they form a flat plane. Now let your thumb stick out in the direction perpendicular to that plane. The +x axis runs out along your thumb, the +y axis runs out along the forefinger, and the +z axis runs in the direction of the middle finger. If you have never tried this, do so; using your left hand gives the opposite direction for +z! There are other ways of describing the right-hand rule, but I have found this description to be the easiest for students to reproduce.

A three-dimensional, right-handed system is usually drawn in one of two ways. When first introduced, x is usually kept going to the right, and y is then kept going up. This then requires that the z axis comes out of the page. This is typically drawn in perspective. However, for the more advanced students who need to do transformations between coordinate systems, the system is usually drawn with the xy plane being horizontal, and the z axis being vertical, as shown in the figure.



We have designed the CARDIS such that the xy plane is horizontal on the table, and that the z axis is vertical, as usually depicted in advanced calculus tasks. If the learners have trouble with the fact that x and y are not represented as horizontal and vertical, respectively, it would be possible to orient the CARDIS such that the xz plane lies on the table, and the foam sheet will represent the xz plane rather than the xy plane. However, in this case it will be necessary to add an extra piece of foamboard permanently fixed in the xy plane. Otherwise marking the points becomes difficult, as gravity is no longer working in your favor.

The CARDIS axes have been labeled with tactile tickmarks every 5 cm, and have been labeled in Braille. The origin is represented by the small hole drilled in the pipe fitting; the r vector is represented by a knotted string which starts at that origin, and can be pinned either to the xy plane or to a vertical plane. A sheet of foam serves as the xy plane. The plane has been marked with gridlines that line up with the tickmarks on the axes. There is a vertical plane made of foamboard; this plane is clipped to the +z axis and is free to swing back and forth above the xy plane. It can be used to represent the xz plane, the yz plane, or any vertical plane between those two extremes. This plane is also marked with gridlines that line up with the tickmarks on the axes.

The position of the r vector is demonstrated by pinning the cord to the vertical plane. The pin now represents a point in space, which is a three-dimensional vector. The vector can be described by either three Cartesian coordinates, or three spherical coordinates. Let us call it the vector-point. In order to describe the different coordinates, it is also useful to describe two additional points, one of which can be marked with the string.

From the vector-point, pulling the remaining cord straight down perpendicular to the xy plane will intersect the xy plane at x and y coordinates of the r vector. The cord can now be pinned to the xy plane at that point, so that the learner can measure the x and y coordinates. Let's call it point A, which has the coordinates x_A, y_A and z = 0. The distance from the origin to that point is r_A = the square root of the sum of the squares of x_A and y_A, as required by the Pythagorean theorem. The angle in the xy plane, as measured from the +x axis and sweeping towards the +y axis until it meets the segment r_A, is the azimuthal angle theta (θ). On CARDIS each ten degrees of azimuthal angle is marked with cord of a different texture, and labeled in Braille.

To find the second point of interest, draw a second line. This line is horizontal, parallel to the xy plane, and goes through both the vector-point and the z axis. With CARDIS, this line can be felt by feeling along the vertical plane from the vector-point inwards towards the +z axis, parallel to the xy plane. The point at which this line intersects the +z axis at the z coordinate of the vector-point; let's call this point B. This point has the coordinates x = 0, y = 0, and z = z_B. The angle between the +z axis and the vector r, sweeping down from the +z axis to the vector, is phi (φ), the polar angle. On CARDIS each ten degrees of azimuthal angle is marked with cord of the same texture as used for theta, and have also been labeled in Braille.

In equation form, the conversions between Cartesian and Spherical coordinates are:

$x = r\ sin\varphi\ cos\theta$	x equals r times sine phi times cosine theta
$y = r\ sin\varphi\ sin\theta$	y equals r times sine phi times sine theta
$z = r\ cos\varphi$	z equals r times cosine phi
$r = \sqrt{x^2 + y^2 + z^2}$	r = the square root of the sum of x squared, y squared, and z squared
$\theta = arctan\left(\frac{y}{x}\right)$	theta = arctangent of the quantity y divided by x

$\varphi = arccos\left(\frac{z}{r}\right)$	phi = arccosine of the quantity z divided by r

In order to allow learners to make measurements with CARDIS and have them come out reasonably accurately, we recommend that instructors align the ***r*** vector on angle gridlines for both theta and phi. Otherwise it will be difficult to estimate the angle to any better than plus or minus 5 degrees. It is possible to estimate the Cartesian coordinates either to the nearest half or nearest third of a unit. Using this method, we were able to achieve better than 5% accuracy, as shown below. In the future we hope to do a rigorous analysis of the uncertainties that can reasonably be achieved without preferential alignment as just described.

Measured	**Calculated from above equations**
r = 10	r = 9.9
theta = $20° = \theta$	theta = $21° = \theta$
phi = $50° = \varphi$	phi = $49° = \varphi$
x = 7	x = 7.2
y = 2.7	y = 2.6
z = 6.5	z = 6.4

Suggestions, Questions, and Class Exercises

The construction manual for CARDIS will be posted at http://astrosense.astro4dev.org/. An analysis of the uncertainties involved in making measurements is also planned. We welcome feedback and suggestions; please email kathy.eastwood@gmail.com. Class exercises are being developed, but will not be disseminated until they have been tested in the classroom.

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